ECLIPSING BINARY STARS: PAST, PRESENT, AND FUTURE

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Abstract

A brief history of eclipsing binary star light curve analysis and an overview of the current state of affairs is given. The photometric searches for extra-solar planets in the next few years will eventually result in millions of light curves and such volumes of data will require the efforts of amateur astronomers in their analysis. This article is a thinly veiled plea to AAVSO members to join the current efforts of a few AAVSO members in the observation and analysis of eclipsing binary stars.

1. Introduction

The 20th century saw tremendous advances in the field of stellar astrophysics. At its beginning, observational surveys (photographic and spectroscopic) had begun to reveal the properties of stars, but it took many developments—such as quantum mechanics—to fully understand the nature of these observations. It was clear that a substantial number of stars were actually binary systems and a few of those had orbits inclined at an angle that caused the stars to pass in front of one another as seen from Earth. The value of these eclipsing binaries in determining the properties of stars, especially masses and radii, was recognized early on and some very clever methods were developed to analyze binary star data in a time when mechanical calculators were a novelty and the basic physics behind modern computers hadn't even been developed. Eclipsing binaries became veritable astrophysical laboratories and received considerable attention from observers and theoreticians.

Today, interest in eclipsing binaries among professional astronomers has declined, primarily, I think, because there is a widespread (and mistaken!) belief among those outside our field that all of the interesting problems have been solved and the remaining work is merely tidying up. On the contrary, there are many theoretical and observational areas that remain practically unexplored, mysteries certainly less grandiose than figuring out the origin of the Universe but nonetheless interesting, challenging, and important.

We are about to see a resurgence of interest in binary stars, driven by advances made on the observational front: optical interferometers and large-scale photometric surveys. With the optical interferometers now operating or nearing completion, we will be able to resolve many more interacting binaries and complement our current photometric and spectroscopic data, allowing us to answer questions that currently remain out of reach. Driven by the interest in detecting transits of extra-solar planets, large-scale photometric surveys will monitor huge numbers of stars for photometric variability, revealing thousands of new eclipsing binaries. To maximize our return on these projects, we will have to expand the community of people knowledgeable in the analysis of the data and provide them with better tools for that analysis. Amateur astronomers will play a crucial role in collecting and analyzing eclipsing binary star data in the coming years.

2. In the beginning

The history of binary star light curve analysis can be split into two eras: the era of data rectification and the era of physical modeling, with the boundary drawn by the development of electronic computers. Before the time of high-speed computers, observations of real binaries were “corrected” to remove annoyances such as ellipsoidal variation and the reflection effect and then modeled by spherical stars whose properties could be more easily computed with specially prepared graphs. About 1970, the field did an about-face and rather than forcing the data to match a simplified model, the models were made more sophisticated so as to match the observations directly.

The early days of binary star light curve analysis were dominated by the contributions of Henry Norris Russell (Russell 1912a, 1912b, 1939, 1942, 1945, 1948; Russell and Merrill 1952) of Hertzsprung-Russell diagram fame. Russell was primarily interested in estimating the parameters of the stars in an eclipsing binary and developed what we now refer to as the Russell model with increasing sophistication over several decades. Nowadays, “sophisticated” is not what comes to mind when we look at the Russell model with its similar ellipsoids for the star shapes and rather crude treatment of tidal and mutual heating effects. But to label the Russell model as “crude” would be unfair, or at least misleading, because even though we know much more about the physics of binary stars today, we would be hard pressed indeed to come up with a better model if we were constrained by the lack of computing power that Russell faced. Russell was after real results, not theoretical adventures, and reading his papers reveals the pure cleverness of his model.

As the Russell model was reaching its developmental apex in the 1940s and 50s, Zdeněk Kopal was thinking ahead and developing ideas that would later be employed in the physical models of today’s light curve programs. These contributions are detailed in his book *Close Binary Systems* (Kopal 1959) and include the use of the Roche model equipotentials to model the surface figures of the stars and the morphology of binaries based on the Roche model. Direct use of the Roche model in analyzing light curves was still prohibitive computationally: Kopal developed a method of correcting the light of a spherical model for Roche model distortions (Kopal 1942). As Russell (1948) described it:

> This difficult problem has been solved by Kopal. His method permits the calculation of the observed light received from a given system at any phase, given its fundamental physical properties.
>
> The formulae involve series expansions and many new functions of the various arguments. To invert them and obtain a direct solution for the elements from the observations is hopeless.

Hopeless indeed in a time when computers were just coming onto the scene.

The field seemed to be static until the late 1960s, when the availability of electronic computers became more common. Suddenly the intractability of using the Roche model to compute directly the shapes of the components of a close binary was eliminated. We could now eliminate the necessity of using similar ellipsoids and model the shapes of the stars with surfaces of constant potential energy (equipotentials), the shapes we expect stars in hydrostatic equilibrium to have. The first person to apply the Roche model directly in a light curve synthesis code was Lucy (1968), who applied the Roche model to the W UMa stars. W UMa systems are very common, late-type binaries whose components are so close together as to be touching one another, and are thus known as overcontact binaries. Obviously, one would have absolutely no hope of deriving anything meaningful from an application of the Russell model to overcontact systems since the stars are nothing even close to ellipsoids. Lucy's paper marks the shift from rectifying observations to using a physical model to include "annoyances" like ellipsoidal variation in a computed light curve that could be compared directly to observations. Suddenly, these annoyances became sources of information about the stars.

Shortly after Lucy's paper, several new light curve models were published and over the years, two of them have seen substantial use in the binary star community: LIGHT (and its successor LIGHT2, which includes Rucinski's WUMA3 code) by Graham Hill (Hill and Hutchings 1970; Hill, 1979; Hill and Rucinski 1993) and the Wilson-Devinney program (WD; Wilson and Devinney 1971; Wilson 1979, 1990, 1993). Both are based on the Roche model and either one is a fine choice for doing light curve analysis. My subsequent comments, though, will cover the WD program since that is the program I have used over the years. Bob Wilson continues to update and expand the WD program and it is now capable of modeling much more than light curves. It can also model radial velocities (Wilson and Sofia 1976), limb polarization (Wilson and Liou 1993), spectral line profiles (Mukherjee, Peters, and Wilson 1996), and X-ray pulses (Wilson and Terrell 1998).

I have given only the briefest overview of a rich history of eclipsing binary light curve analysis. For a more in-depth review, see Wilson (1994a) and the book *Eclipsing Binary Stars: Modeling and Analysis* by Kallrath and Milone (1999).

3. Where we stand

On the observational side of things, CCD cameras have greatly improved the quality and quantity of photometric observations of eclipsing binaries. Because CCD cameras simultaneously measure multiple objects and the sky background, high-quality observations can be made even in less than optimal conditions. Another advantage is that CCD observations allow the observer better to optimize data reduction since quantities like the measuring aperture don't have to be decided upon at the telescope. They are also quite efficient at converting the incoming photons into a signal that we measure to find the brightness of a star. This efficiency makes it possible for a given telescope to measure fainter stars.

All of these qualities of CCD cameras have helped them make a great impact on the acquisition of eclipsing binary light curves. The most important thing about CCD cameras, however, is that they have become much less expensive. No longer are they solely the domain of astronomers at professional observatories. Amateur astronomers are doing quite amazing things with small telescopes and CCD cameras. As I write this, the cataclysmic variable WZ Sge is in one of its superoutbursts that occur about every 33 years and amateur astronomers around the world are doing high-precision photometry in incredible quantities, literally tens of thousands of observations in the last few weeks. (Unfortunately, many of those are unfiltered observations which make them much less useful for analysis than the filtered ones. Filters separate the radiation coming from different parts of the binary, enabling us to have a better handle on the astrophysical parameters of the system, and this is especially true with systems like WZ Sge, in which there are large amounts of circumstellar matter. So, please do consider using filters. They will make your observations much more valuable.)

In the not so distant past, one would be quite pleased to have a few hundred measurements covering the entire light curve of an eclipsing binary. Nowadays we obtain that many in a single night of observing and when it comes time to analyze the data, we have thousands of quality observations covering the entire light curve, often in multiple filters. So, on the observational side of things amateurs are making tremendous contributions to our understanding of binary stars. But can they do even more? They definitely can.

Today, professional astronomers perform the analysis of binary star light curves, but there is no reason why dedicated amateurs could not analyze their observations to find the parameters of the binary system. Modern PCs have plenty of power to do the necessary computations and the Internet provides for easy communication between experienced users of the light curve analysis codes and amateur astronomers who wish to learn how to perform the analysis. The time is ripe for amateur astronomers to extend their contributions into the domain of light curve analysis. If you would like to learn more about analyzing binary star data, please contact me.

In May 2001 at the Spring Meeting of the AAVSO in Madison, Wisconsin, we held the first of what I hope will be several workshops on light curve analysis.

The workshop covered various classifications of binaries, *e.g.*, based on discovery method (eclipsing, spectroscopic, visual, etc.) or morphological type (detached, semi-detached, overcontact, or double contact) and then narrowed its focus to the light curves of eclipsing binaries. Participants learned about the kinds of information that could be extracted from light curves and were given a brief introduction to the WD program.

4. A quick look at light curve analysis

The essence of light curve analysis is pretty simple: you use a model that has the necessary physics to predict how the light curve of an eclipsing binary should look and compare that predicted light curve with the observed one. By adjusting the parameters of the model, you try to find the light curve that best fits the observed one. But light curve analysis is a bit like chess: the rules are fairly simple but mastering the game requires a lot of hard work and years of experience.

As mentioned previously, WD is based on the Roche model, so a few words on that topic would be appropriate. The classical Roche model is based on a few assumptions:

1. The gravitational fields of the stars behave as though the stars are point masses.
2. The orbits of the stars around their common center of mass are circular.
3. The stars rotate synchronously. That is, they rotate once each orbit.
4. The gas is in hydrostatic equilibrium and thus is not flowing.
5. There are no radiation pressure effects.

The first assumption might at first seem to be very poor, especially for highly distorted stars, but stars are highly centrally condensed and become more so as they evolve. Since most of the mass is contained in the core, the point mass approximation is pretty good. The second assumption means that binaries with eccentric orbits and/or rapid rotation cannot be modeled but the Roche model has been extended to deal with these cases, so they are not real limitations these days. The final assumption works for all but the very hottest stars.

The Roche model is useful because it enables us to study the shapes of the stars and how the stars will interact as they evolve. By establishing a frame of reference that rotates with the binary, we can study surfaces of constant potential energy (equipotentials) to learn about the binary. It isn’t necessary to go into the detailed mathematics of equipotentials to understand them. One needs merely to have taken a bath.

When you pour a certain amount of water into the bathtub, the water settles out to have a level surface rather than piling up at the front of the bathtub under the faucet. This level surface is an equipotential surface and any attempt to raise part of the water to a higher level results in the water’s quick return to the level surface. You can raise the level surface by adding more water to the bathtub, but it will settle out to a new level surface of constant potential energy. The stars in a binary do exactly the same thing.

The surfaces of the stars will coincide with the equipotential surfaces of the binary, and we can generate contour plots that show the shapes of these surfaces. Figure 1 shows a few of the infinite number of equipotential surfaces in a binary in which one star is twice as massive as the other.

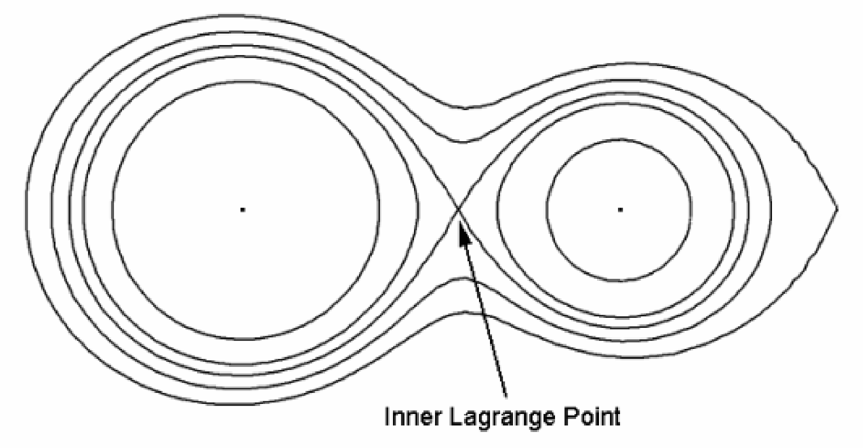

Figure 1. Contours in the orbital plane of the equipotential surfaces in a binary with a 2:1 mass ratio. The more massive star is on the left. The centers of the stars are indicated by the dots.

Note that the equipotentials close to either star are close to being spherical but they become distorted as they near the other star. One equipotential surface plays a crucial role in the evolution of the binary: the equipotential that includes the inner Lagrange point (L_1) point. The volume contained by this special equipotential surface is known as the Roche lobe. At the L_1 point, the gravitational pull towards the one star is exactly balanced by the gravitational pull of the other star plus the centrifugal force due to the rotation of the binary system. It is an equilibrium point, but an unstable one. A small particle placed at L_1 would not remain there. It would quickly be perturbed and then fall toward one of the stars, rather like the way that a ball perched at the peak of a sharp hill would roll away from the top.

The Roche lobe is important because it defines the largest size that either star can achieve. We know that stars go through rather large expansions during their evolution, but a star in a binary cannot expand beyond the Roche lobe (unless the other star has reached its Roche lobe, in which case the stars can then grow larger than their Roche lobes and become an overcontact binary). If it tries to do so, material pours through the L_1 point and falls toward the other star. Going back to the bathtub analogy, the L_1 point is like the overflow hole in the bathtub. Once the water level reaches the overflow hole, the level of the water remains constant because water is going out as fast as it is coming in.

The Roche model thus enables us to classify binary systems based on the sizes of the two components compared to their Roche lobes. If both stars are smaller than their Roche lobes, the binary is said to be *detached*. If one star fills its Roche lobe while the other is smaller than its Roche lobe, the system is *semidetached*. If both stars have reached their Roche lobes, it is possible for both stars to grow larger than their Roche lobes and we have an *overcontact* system. (Returning to the bathtub analogy: the overflow hole has become plugged so now the water level can rise above it.) Overcontact systems have both stars embedded in a common envelope of material. Figures 2 through 4 show the three morphological types based on the classical Roche model.

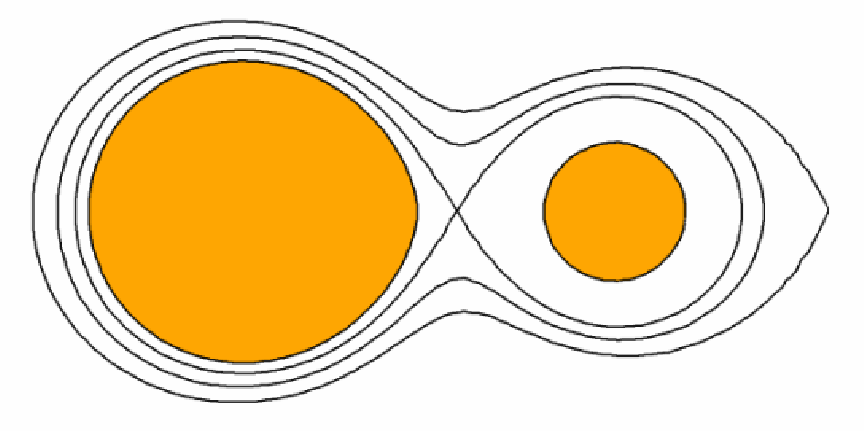

Figure 2. A detached system has both stars smaller than their Roche lobes.

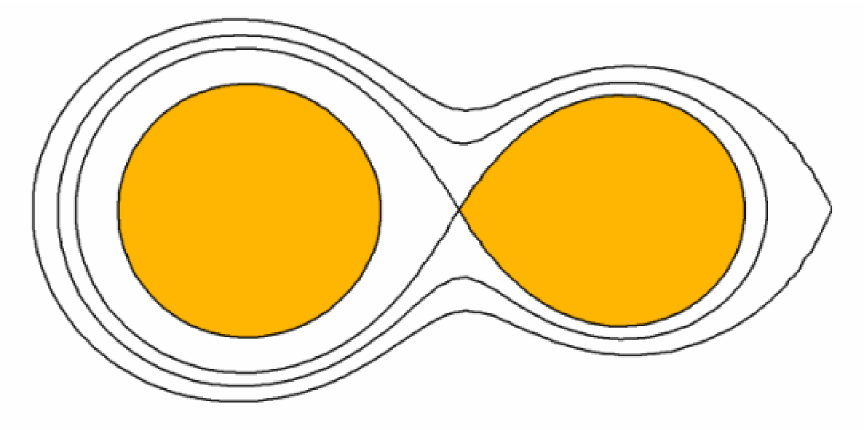

Figure 3. In a semidetached system, one star fills its Roche lobe while the other is smaller than the Roche lobe.

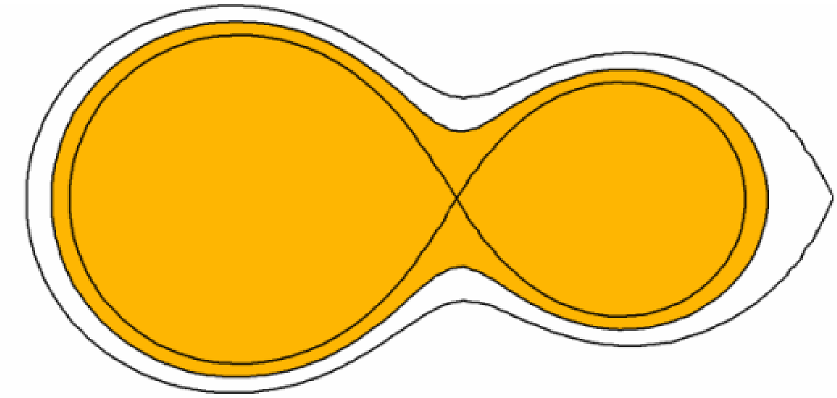

Figure 4. In overcontact systems both stars are contained within a common envelope of material.

In the extended Roche model, eccentric orbits and non-synchronous rotation can be treated (Wilson 1979). The extended Roche model provides for a fourth morphological type that Wilson terms *double contact*. In double contact systems, both stars fill their limiting lobes but at least one of them rotates faster than synchronously. In this case, the shape of the rapidly rotating star is strongly affected by the rotation and it develops an equatorial bulge. Figure 5 shows a computer-generated image of RZ Scuti based on a light curve analysis by Wilson *et al.* (1985).

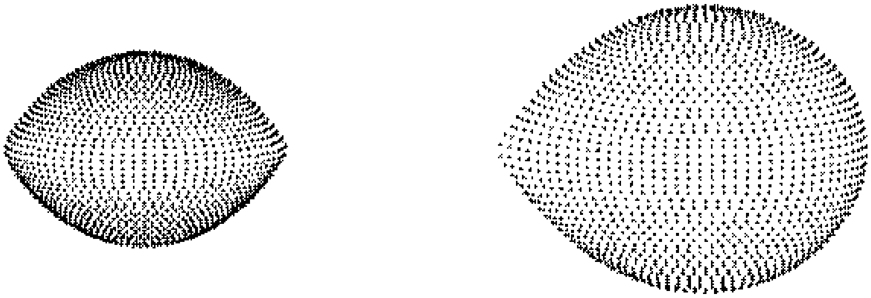

Figure 5. Computer-generated picture of RZ Scuti, a double contact binary. The star on the left is rotating faster than synchronously while the one on the right is rotating synchronously.

So binaries come in many different shapes and sizes and that leads to a wide variety of light curves. Light curves are classified into three groups based on their shapes. Algol-type (EA) light curves are light curves that have deep eclipses with only small variations outside eclipse, as seen in the KK Dra light curve plotted in Figure 6. Algol is the prototype of a class of semidetached binaries in which the cool, low-mass secondary star fills its Roche lobe. Because the secondary is so much cooler than the primary, its surface brightness is much lower and when it is eclipsed, the drop in the total light of the system is small, amounting to a few percent in most Algols. But when the primary is eclipsed, the eclipse is deep because the primary's surface brightness is much higher than that of the secondary.

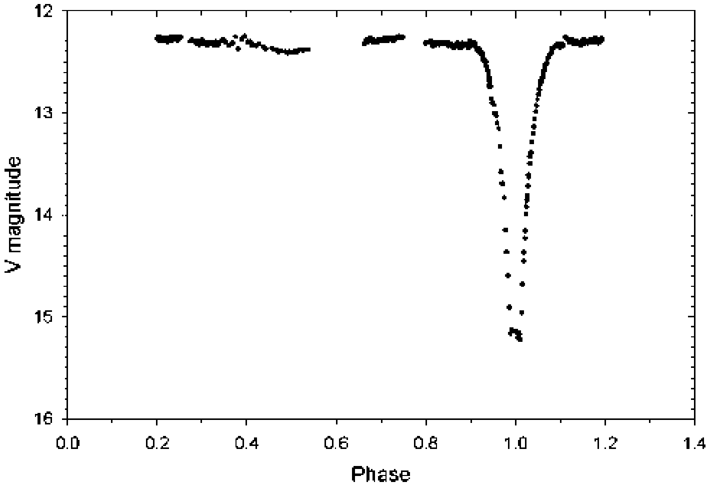

Figure 6. Light curve of KK Dra, a recently discovered Algol-type system. The primary eclipse is nearly three magnitudes deep while the secondary eclipse (at phase 0.5) is barely perceptible. Data are from Guilbault *et al.* (2001).

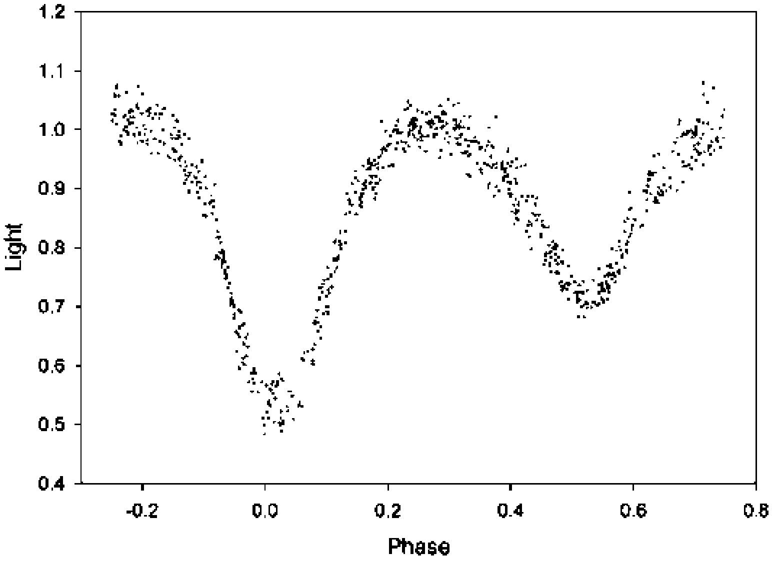

Figure 7. *V* band light curve of β Lyrae. Data are from Van Hamme *et al.* (1995).

The second class of light curves is the β Lyrae type (EB), in which the eclipse depths are noticeably different and the variation between eclipses is dramatic and continuous. Figure 7 shows a light curve of β Lyrae.

The third class of light curves is the W UMa type (EW), in which the eclipse depths are similar and there is continuous variation between the eclipses. W UMa is the prototype of the overcontact binaries. Figure 8 shows the *B* and *V* light curves of the overcontact system V523 Cas. Overcontact systems are quite numerous, but not well understood. Long-term observations in multiple filters will help unravel their mysteries, such as why the light curve maxima are often at different levels (Samec and Terrell 1995; Samec *et al.* 1993).

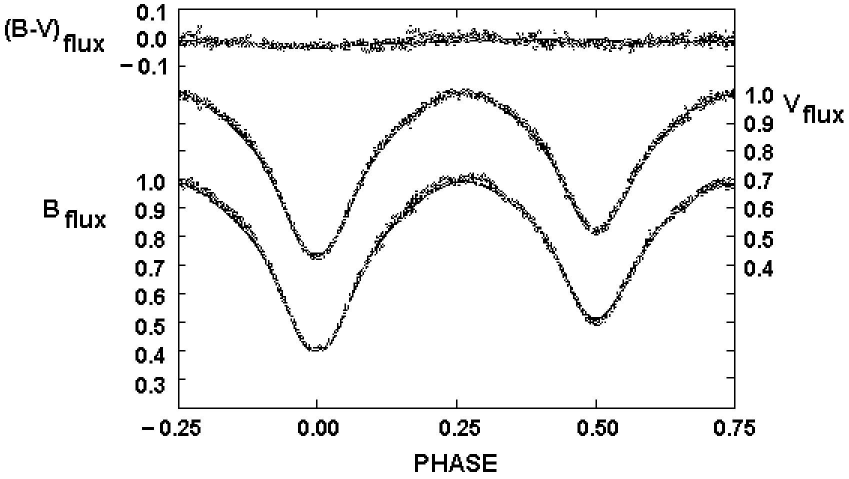

Figure 8. *B* and *V* light curves of V523 Cas showing the shape of a typical EW light curve. The data are from unpublished data kindly sent to the author by Ron Samec (2001).

The classification of light curves based on their appearance goes back to the days when we didn't know much about the physical structure of binaries, and these days is of minimal usefulness. Systems with roughly similar light curves can actually be quite different from each other. I would bet that most EB systems listed in the *General Catalogue of Variable Stars* (Kholopov *et al.* 1985) are not even remotely similar to β Lyrae.

So, you've spent hours collecting data on an eclipsing binary. Now what? It's time to analyze those data and derive physical information about the binary. That's where a program like WD comes in. You spend some time looking at your light curves as well as any additional information on the binary such as spectroscopy, and then estimate a set of parameters. Then you run WD with your best-guess parameters and see if the light curve that WD predicts is anywhere close to the observed light curve. Then you look at the discrepancies between the observed and computed light curves and adjust the parameters to try to correct for those deficiencies based on your understanding of how various parameters affect the light curve.

For example, your computed curve might compare well with the observed one almost everywhere except that the eclipse depths are too shallow. The answer is probably that the inclination you used is too low if the eclipses are partial. Increasing the inclination will increase the depths of the eclipses.

Sometimes, though, it's not so obvious which parameter to adjust because some parameters have similar effects on the light curve. Changing one parameter can produce the same or nearly the same result as changing another parameter. In cases like this, the parameters are said to be highly correlated and you will need additional information to break the correlation. In the case of radial velocity curves, for example, the semi-major axis (a) of the binary orbit is perfectly correlated with the inclination (i). You can cause exactly the same change in the computed radial velocity by increasing the inclination or by increasing the semi-major axis. In that case you can't determine both parameters, merely a combination of them: a sin(i).

If the spectroscopic binary is also an eclipsing binary, however, you can break the correlation because the inclination affects the light curve but the semi-major axis has no effect on it. If you have the inclination set to a value that correctly reproduces the eclipses, then you know that adjustments necessary to match the radial velocity curves must be made to the semi-major axis. Light and velocity curves by themselves give us incomplete information: the light curve can give us the inclination but nothing about the semi-major axis, while the radial velocity curves can give us only the product of the two. So, by combining the light curves with radial velocity curves, you can find a more comprehensive solution for the parameters of the binary.

Just as multiple types of observations can improve the solution, *constraints* can also help. One example of a constraint is the duration of the X-ray eclipse in an X-ray binary. In such systems the X-rays arise from the accretion of matter onto a neutron star, and when the optical star eclipses the neutron star, we see the eclipse in the X-ray light curve. Since the neutron star is essentially a point source, the width of the X-ray eclipse is determined by the size of the optical star. By forcing our (optical) light curve solution to have a set of parameters that reproduces the observed X-ray eclipse width, we are applying a constraint. If the X-ray eclipse lasts for two hours, for example, it doesn't matter how well our solution fits the optical light curve if it requires an X-ray eclipse duration of four hours. We know that set of parameters can't be right even though it produces a light curve that matches the observed one pretty well.

There are many parameters involved in analyzing binary star light curves but most of them have small effects on the light curve and can be set to their theoretical values so that one is usually adjusting only a handful of parameters like the inclination, temperature of one star, sizes of the stars, etc.

Once we manually make the computed light curve match the observed one reasonably well, we can use the parameter adjustment capabilities of WD to zero in on the best fit. Currently, WD uses the method of differential corrections (DC) to minimize the differences between the observed and computed curves. The difference between an observation and the computed value is known as a residual. DC attempts to minimize the sum of the squares of the residuals.

You might wonder why we don't just start with any random set of parameters and let DC find the right solution. The answer is multi-faceted. First, there is more to light curve analysis than adjusting parameters. There is no substitute for a human when it comes to seeing something peculiar and potentially interesting as a solution progresses. Programs like WD are sophisticated but they don't have the creativity and flexibility of an experienced astronomer that would enable them to interpret some unexpected phenomenon. Secondly, correlations, as noted earlier, cause ambiguities and an experienced astronomer can guide the solution away from physically unrealistic regions of parameter space.

Light curve analysis can be tricky and difficult, but the combination of the experience and insight of a human with a sophisticated analysis program like WD can give us a reliable set of parameters for an eclipsing binary. There is no reason why amateur astronomers can't develop that insight and experience and take their scientific contributions to the next level by performing the analysis of their data. One needn't know the gory details of the innards of a program like WD to become proficient in its use. And if we are to get the most out of the coming avalanche of data on eclipsing binaries, amateur astronomers are going to have to play a larger role than they do now.

As with the section on the history of light curve analysis, I have given only the briefest overview of how light curve analysis works. A proper treatment would easily fill a very hefty book but Bob Wilson has written a nice overview of the process (Wilson 1994b).

5. The next few years

The Simbad catalog (http://simbad.u-strasbg.fr) currently lists over 4000 eclipsing binaries of the EA, EB, and EW types. That number will increase tremendously over the next couple of decades, perhaps approaching 100,000 systems, as large-scale surveys are undertaken. In fact, we are already seeing the results of smaller-scale surveys and they are impressive. The Robotic Optical Transient Search Experiment (ROTSE) was designed to look for the optical counterparts to gamma ray bursts, but in the process it has discovered over 500 eclipsing binaries in a survey covering about 5% of the sky area that it monitors. A group of us in the AAVSO are busy doing follow-up studies of some of the more interesting ROTSE discoveries (*e.g.*, Billings *et al.* 2001; Lubcke *et al.*2000).

A hot topic these days is the search for the transits of extra-solar planets across the disks of the stars they orbit. The discovery of the transit of a planet across the disk of HD 209458 (Charbonneau *et al.* 2001; Henry *et al.* 2000) has heightened the interest in photometric searches for planetary transits. Space-based missions like the proposed NASA Discovery mission Kepler (Borucki *et al.* 1997) and ESA's Eddington mission have as primary mission objectives the search for extra-solar planets by monitoring tens of thousands of stars with high photometric precision, but they will also discover thousands of eclipsing binaries in the process. And, as the Hipparcos satellite showed, astrometric missions like FAME and GAIA will also discover large numbers of eclipsing binaries. Mission designers expect GAIA to provide several million high-precision light curves—a staggering amount of data.

We will have to increase the efficiency of our analysis tools and the number of people who can obtain and analyze data on these binaries. For us to utilize these data fully, amateur astronomers will have to play a critical role in eclipsing binary research. We need more people who can obtain and analyze high precision (1%) photometry. The AAVSO is paving the way for interested people to obtain the training they need. The eclipsing binary workshop held at the Spring 2001 meeting was merely a first step. At the Fall 2001 meeting, Arne Henden of the USNO Flagstaff station will lead a workshop on CCD photometry, and there is already discussion of holding multi-day workshops on photometry and data analysis in the near future.

Binary stars are important for many reasons. They provide valuable mass, radius, and luminosity data on stars, for example. But they are fascinating beasts in and of themselves. At the workshop I presented a list of interesting binaries:

1. *Algols*—are Algols with rapidly rotating primaries an evolutionary link to the W Ser systems?
2. *W Ser systems*—longer period systems either in or just past a phase of rapid mass transfer. Are they precursors to Algols? What is the structure of the accretion disk?
3. *W UMa systems*—overcontact binaries. What is their internal structure? What causes them to exhibit short-term variability in their light curves? Why do a large number of them show asymmetries between the two maxima in the light curves (the O'Connell Effect, see Davidge and Milone 1984)? How do they form?
4. *ε Aurigae*—an eclipsing binary with a 27-year period consisting of an early F supergiant and a ... something. The best guess is that the secondary is slightly tilted disk (Wilson 1971) with a central opening cleared out by an embedded binary (Eggleton and Pringle 1985; Lissauer and Backman 1984), making ε Aur a triple system.

And there are many others. Amongst the soon-to-be-discovered eclipsing systems, is there another ε Aurigae or β Lyrae? Will we discover systems just as perplexing and yet very different from anything we've seen? I have no doubt that we will. We have barely begun to understand the origin and evolution of interacting binary stars, and over the next few decades, we will make great progress on that front, powered in no small part by the efforts of talented and dedicated amateur astronomers.

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